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On the Jeans instability during the QCD phase transition

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Abstract

The Jeans scale is estimated during the coexistence epoch of quark-gluon and hadron phases in the first-order QCD phase transition. It is shown that, contrary to previous claims, reduction of the sound speed is so little that the phase transition does not affect evolution of cosmological density fluctuations appreciably.

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As is well known, the Jeans length scale, above which density fluctuations can grow, is given by the sound speed c_s multiplied by the dynamical time scale. In the radiation-dominant stage of the early universe, c_s is equal to $3^{-1/2}$ times the light speed, which we take to be unity here, so density fluctuations can grow only on super-horizon scales.

Recently, however, Jedamzik [1] and, independently, Schmid, Schwarz and Widerin [2] proposed that if a cosmological quark-hadron phase transition is of first-order, which is supported by some of the recent lattice simulations[3], the sound speed effectively vanishes during the coexistence period of the quark-gluon and the hadron phases. If this were true, the Jeans scale would become vanishingly small, and even sub-horizon-scale fluctuations should grow during this epoch. This temporal decline of the Jeans length would imprint a nontrivial feature on the processed spectrum of fluctuations around the solar-mass scale. [2] Furthermore, it has been suggested that such anomalous growth of perturbations could induce efficient formation of primordial black holes (PBH's)[1]. The resultant mass function of the PBH's would have a peak on the solar-mass scale which is about the scale of MACHO's. Thus the idea raised in Refs. 1) and 2) is very interesting and may be astrophysically important. Unfortunately, however, the actual evolution of the sound speed, as well as that of the Jeans length, has not been given in either paper, and their claims are limited to a qualitative level.

In the present paper we present the result of an explicit calculation of the sound speed using the bag model[4]. We first review how the quark-hadron phase transition proceeds in this model and then calculate the sound speed under two distinct conditions.

In the bag model [4, 5] the energy density of the quark-gluon phase, ρ_q, is the sum of that of the relativistic particles and the QCD vacuum energy density, namely, the bag constant, B.

$$\rho_q(T) = \frac{\pi^2}{30} g_q T^4 + B \ , \tag{1}$$

where g_q is the effective degree of relativistic freedom. $g_q \cong 51.25$ when the number of

species of relativistic quarks is two, and $g_q \cong 61.75$ when it is three. The pressure, p_q, and the entropy density, s_q, are given by

$$p_q(T) = \frac{\pi^2}{90} g_q T^4 - B \ , \tag{2}$$

$$s_q(T) = \frac{4\pi^2}{90} g_q T^3 \ , \tag{3}$$

respectively. On the other hand, the energy density, ρ_h, the pressure, p_h, and the entropy density, s_h, of the hadron phase are expressed as

$$\rho_h(T) = \frac{\pi^2}{30} g_h T^4 \ , \tag{4}$$

$$p_h(T) = \frac{\pi^2}{90} g_h T^4 \ , \tag{5}$$

$$s_h(T) = \frac{4\pi^2}{90} g_h T^3 \ , \tag{6}$$

where $g_h \cong 17.25$ is the degree of relativistic freedom in this phase. At the coexistence temperature, T_c, we have $p_q(T_c) = p_h(T_c)$, and hence it is related to the bag constant by

$$B = \frac{\pi^2}{90} \left(g_q - g_h \right) T_c^4. \tag{7}$$

In this model, the phase transition from the high-density quark-gluon phase to the low-density hadron phase starts with bubble nucleation, whose probability per unit time per unit volume, $P(T)$, is given by [5]

$$P\left(T\right) \simeq C T_c^4 \exp\left(-\frac{16\pi}{3} \frac{\sigma^3}{T_c L^2 \eta^2} \right) \ , \tag{8}$$

where L is the latent heat per unit volume of the phase transition, σ is the free energy per unit surface area of the bubble, C is a constant, and η, which represents the degree of super cooling, is defined by

$$\eta \equiv \frac{T_c - T}{T_c} \ . \tag{9}$$

The exponent of $P(T)$ is singular at T_c, and $P(T)$ vanishes then, but it increases drastically when T drops slightly from T_c.

Hence the phase transition is expected to proceed as follows. In the course of the cosmic evolution the cosmic temperature decreases, and until $T = T_c$ the entire universe is occupied by the quark-gluon phase. At this temperature, the hadron phase does not appear yet, and the super cooling ($T < T_c$) occurs. But only a small degree of super cooling, $\eta \sim 10^{-3}$, actually takes place since $P(T)$ soon becomes large[5]. Then the nucleation of the hadron phase starts, and the universe is reheated by the latent heat up to T_c so that the pressure equilibrium between two phases is realized and subsequent nucleation is suppressed. Afterwards, the phase transition proceeds through expansion of nucleated bubbles whose speed is small and estimated to be around 10^{-3}[6], keeping the balance between heating due to continuous liberation of latent heat and cooling caused by cosmic expansion. The temperature and the pressure remain constant in this stage, but the mean energy density,

$$\bar{\rho} \equiv f_q \rho_q + (1 - f_q)\, \rho_h \ , \tag{10}$$

decreases gradually from $\rho_q(T_c)$ to $\rho_h(T_c)$ as the volume fraction of the quark-gluon phase, f_q, drops. During this epoch the scale factor expands by a factor of about $(g_q/g_h)^{1/3} \simeq 1.4$.

It is in this coexistence regime that the authors of Refs. 1) and 2) suggested the sound speed vanishes. According to Ref. 1), an adiabatic compression of the system in the equilibrium two-phase mixture induces conversion of the low-energy hadron phase into high-energy quark phase, and then the mean energy density grows due to the change of f_q, with the pressure remaining constant. This implies that pressure response to a change in energy density is anomalously small, and hence, by their argument, the sound speed should effectively vanish.

In order to clarify whether such an intuitive discussion is correct, we must first find a formula for the sound speed in an inhomogeneous medium consisting of the mixture of two different phases. While such a problem has not been fully discussed in any cosmological context, it is a rather familiar issue in the field of engineering, e.g., propagation of sound

waves in a water-vapor system in a boiler etc[7].

In the rest frame of the sound wave front[†], suppose that the medium ahead of the wave front is characterized by a pressure p, the temperature T, energy densities ρ_q and ρ_h, volume fraction of the quark-gluon phase f_q, and flow velocity c_s, and that the medium behind it is characterized by $p + dp$, $T + dT$, $\rho_q + d\rho_q$, $\rho_h + d\rho_h$, $f_q + df_q$, and $c_s + du$, respectively. Then the continuity equation reads

$$\begin{aligned} & f_q\,(p+\rho_q)\,\gamma^2 c_s + (1-f_q)\,(p+\rho_h)\,\gamma^2 c_s \\ = & \,(f_q+df_q)\,(p+\rho_q+dp+d\rho_q)\,\tilde{\gamma}^2\,(c_s-du) \\ + & \,(1-f_q-df_q)\,(p+\rho_h+dp+d\rho_h)\,\tilde{\gamma}^2\,(c_s-du)\ , \end{aligned} \tag{11}$$

per unit area, where $\gamma = (1-c_s^2)^{-1/2}$ and $\tilde{\gamma} = \left\{1-(c_s-du)^2\right\}^{-1/2}$ are the Lorentz factor [‡]. The momentum equation is given by

$$\begin{aligned} & p + f_q\,(p+\rho_q)\,\gamma^2 c_s^2 + (1-f_q)\,(p+\rho_h)\,\gamma^2 c_s^2 \\ = & \,p + dp + (f_q+df_q)\,(p+\rho_q+dp+d\rho_q)\,\tilde{\gamma}^2\,(c_s-du)^2 \\ + & \,(1-f_q-df_q)\,(p+\rho_h+dp+d\rho_h)\,\tilde{\gamma}^2\,(c_s-du)^2 . \end{aligned} \tag{12}$$

From (11) we find

$$du = c_s \frac{1-c_s^2}{1+c_s^2} \frac{f_q\,(dp+\rho_q) + (1-f_q)\,(dp+d\rho_h) + (\rho_q-\rho_h)\,df_q}{f_q\,(p+\rho_q) + (1-f_q)\,(p+\rho_h)} = c_s \frac{1-c_s^2}{1+c_s^2} \frac{dp+d\bar{\rho}}{p+\bar{\rho}}\ . \tag{13}$$

Inserting this into (12), we obtain

$$c_s = \left(\frac{d\bar{\rho}}{dp}\right)^{-\frac{1}{2}}\ . \tag{14}$$

[†] Since the time scale of QCD is much shorter than the cosmic expansion scale, the following discussion without the effect of cosmic expansion applies to our problem unchanged.

[‡]We thank Dr. Schwarz for pointing out that the nonrelativistic equations were employed in the original version of our paper.

Thus the sound speed in a mixture is obtained by simply replacing the energy density ρ with the mean value $\bar{\rho}$ in the usual formula for a single-component system,

$$c_s = \left(\frac{dp}{d\rho}\right)^{\frac{1}{2}} . \tag{15}$$

The pressure derivative of $\bar{\rho}$ is written by

$$\frac{d\bar{\rho}}{dp} = f_q \frac{d\rho_q}{dp} + (1 - f_q)\frac{d\rho_h}{dp} + (\rho_q - \rho_h)\frac{df_q}{dp} . \tag{16}$$

When the sound speed in the hadron phase and the quark-gluon phase equals to that in the pure radiation, the sum of the first and second terms of the equation (16) provides the value of $3^{-1/2}$. Hence the difference from the single-component system is given by the third term and since $(\rho_q - \rho_h)$ is positive, if the volume ratio of the quark phase increase as the pressure becomes higher, the sound speed in the complex phase should be reduced. In order to evaluate the degree of reduction, the estimation of df_q/dp is necessary.

The next question is under what conditions we should calculate the derivative (14). Usually one evaluates the sound speed under adiabatic conditions, because a change of the entropy associated with sound-wave propagation is small in most cases of interest. Since this is also the condition Jedamzik proposed to adopt[1], let us first calculate the sound speed under this condition.

The isentropic condition reads

$$\begin{aligned} f_q s_q \gamma c_s + (1 - f_q)\, s_h \gamma c_s &= (f_q + df_q)(s_q + ds_q)\,\tilde{\gamma}\,(c_s - du) \\ &\quad + (1 - f_q - df_q)(s_h + ds_h)\,\tilde{\gamma}\,(c_s - du) \ . \end{aligned} \tag{17}$$

Then using the average entropy density, $\bar{s}$, defined by

$$\bar{s} = f_q s_q + (1 - f_q)\, s_h \ , \tag{18}$$

and combining this with the continuity equation (11), we find

$$\frac{d\bar{\rho}}{p + \bar{\rho}} = \frac{d\bar{s}}{\bar{s}} . \tag{19}$$

Using the relation $\bar{\rho} = \bar{s}T - p$, we can see this is nothing but the second law of thermodynamics which gives us no information about df_q/dp. The adiabatic condition is useless for the purpose of calculating the sound speed qualitatively. Hence we must find a more appropriate one, understanding the dynamics of the phase transition better.

What is often adopted in the literature [7] apart from the isentropic condition is the conservation of the *quality* parameter, x, which is the energy fraction of the gas component. The theoretical sound speed in a water-vapor system evaluated under this condition agrees with experimental values qualitatively[7]. In the present case the corresponding quantity may be defined by the energy fraction of the high-energy quark-gluon phase, x_q, as

$$x_q = \frac{f_q \rho_q}{f_q \rho_q + (1 - f_q)\,\rho_h} \ , \tag{20}$$

with which df_q/dp is expressed as

$$\frac{df_q}{dp} = \frac{f_q\,(1-f_q)}{\rho_h}\frac{d\rho_h}{dp} - \frac{f_q\,(1-f_q)}{\rho_q}\frac{d\rho_q}{dp} - \frac{f_q\,(1-f_q)}{x_q\,(1-x_q)}\frac{dx_q}{dp} \ . \tag{21}$$

Since we cannot measure the sound speed experimentally in our case unlike in a water-vapor system, we must work out the appropriate value of dx_q/dp from a theoretical view point alone. Here we propose to calculate the sound speed under the condition $dx_q/dp = 0$. This is appropriate because the transition of the phases through bubble nucleation is totally suppressed at the coexistence temperature, as seen in (8), and the expansion speed of bubbles is so small that energy transfer through bubble expansion or contraction is also expected to be negligible during sound-wave propagation.

Therefore we substitute this condition into (16) and find

$$\left.\frac{d\bar{\rho}}{dp}\right|_{x_q} = 3\bar{\rho}\left(\frac{f_q}{\rho_q} + \frac{1-f_q}{\rho_h}\right) \ . \tag{22}$$

We thus obtain the sound speed under the constant-quality condition as

$$c_s = \frac{1}{\sqrt{3}}\left\{\left(y + \frac{1}{y} - 2\right)\left[-\left(f_q - \frac{1}{2}\right)^2 + \frac{1}{4}\right] + 1\right\}^{-\frac{1}{2}} \ . \tag{23}$$

Here y is defined by

$$y \equiv \frac{\rho_q(T_c)}{\rho_h(T_c)} = \frac{4g_q - g_h}{3g_h}, \tag{24}$$

where we have used (7). Since $y + 1/y \geq 2$, we find from (23) that the sound speed in the mixed state is indeed smaller than that in the case of pure radiation and it can be arbitrarily small if y is much larger or much smaller than unity. In the present case, however, the actual value of y is given by $y = 3.63$ for $g_q = 51.25$ and $y = 4.44$ for $g_q = 61.75$. Hence reduction of the sound speed is mild, and even the minimum sound speed, which is realized at $f_q = 0.5$, is as large as

$$c_s^{\min} = (0.77 \sim 0.82) \times \frac{1}{\sqrt{3}} \tag{25}$$

for $g_q = 61.75$ and 51.25, respectively.

Thus the intuitive claim that $c_s \approx 0$ is proved to be inappropriate as long as we consider the sound speed at the coexistence temperature. Although it is true that the coexistence of different phases at the first order phase transition reduces the velocity of the sound wave, its degree in the case of the quark-gluon to hadron QCD phase transition is insignificant and has no drastic effect on the development of cosmological density perturbations.

Turing back to the original expression (16), we see that the only way to realize a vanishingly small sound speed is to have an extremely large value of df_q/dp. But this is not possible at the coexistence temperature T_c since the nucleation rate (8) vanishes then. Nevertheless it may be possible to realize a large value of df_q/dp at a smaller temperature when the nucleation rate rises significantly, and the sound speed may become vanishingly small then. Unfortunately, however, in the actual evolution of the universe, in the bag model the degree of super cooling is expected to be very small, as mentioned above, so even if the sound speed vanished in this era, the duration of this era is too short to leave any trace in the spectrum of density fluctuations.

Until now we have assumed that the QCD phase transition is of first order and subcritical fluctuations are negligible. If, on the contrary, it were of second order, and subcritical

fluctuations were effective, the phase transition would proceed while maintaining an equilibrium population between quark-gluon and hadron phases. Under such a circumstance, f_q would change only mildly against perturbation by a sound wave. Thus df_q/dp cannot take a large enough value to suppress the sound speed drastically in these cases.

We therefore conclude that the quark-hadron phase transition has no appreciable effect on the final spectrum of density fluctuations.

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References

[1] K. Jedamzik, Preprint astro-ph/9605152.

[2] C. Schmid D. J. Schwarz and P. Widerin, Phys. Rev. Lett. **78** (1997), 791.

[3] See, e.g., K. Kanaya, Nucl. Phys. **B** (Proc. Suppl.) **47** (1996), 144.

[4] A. Chodos, R. L. Jaffe, K. Johnson, C. B. Thorn and V. F. Weisskopf, Phys. Rev. **D9** (1974), 3471.
A. Chodos, R. L. Jaffe, K. Johnson and C. B. Thorn, Phys. Rev. **D10** (1974), 2599.
T. DeGrand, R. L. Jaffe, K. Johnson and J. Kiskis, Phys. Rev. **D12** (1975), 2060.

[5] G. M. Fuller, G. J. Mathews and C. R. Alcock, Phys. Rev. **D37** (1988), 1380.

[6] K. Kajantie, Phys. Lett. **B285** (1992), 331.

[7] R. E. Henry, Chem. Engr. Prog. Symp. Ser. 66-102 (1970), 1.
K. Akagawa, *Gas-Liquid Two-Phase Flow* (Corona Publishing, Tokyo, 1974).